NOTATION

u(x), distribution curve for moisture content over sample height; x, coordinate measured along height of sample with x = 0 at bottom of sample holder; ρ_0, sample density; G, G_1, G_2, moisture flow densities; a_m, moisture diffusion coefficient; δ, moisture thermodiffusion coefficient; τ, absolute value of temperature gradient; t, time; Θ, temperature, L, sample height.

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OPTIMUM CONTROL OF PROCESSES OF HEAT AND MASS TRANSFER DURING DRYING

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The problem of the optimum control of processes of conjugate heat and mass transfer during drying is solved for the case when the controlling parameter is the temperature of the drying agent and the quality criterion is the heat expenditure.

Despite the fact that the structure of the system of differential equations describing the processes of heat and mass transfer during drying is well known [1], the use of mathematical methods of optimization of drying processes is complicated by a number of factors, among which one must include the absence of analytical expressions for the coefficients of the equations which are functions of the parameters of the drying process and the absence of sufficiently reliable methods of optimization. Even if analytical expressions for the coefficients of the system are known, their substitution into the equations of the system so complicates the latter that one is often not able to use exact methods of optimization. The replacement of the variable coefficients of the system by constants reduces the accuracy of the mathematical description of the process and in a number of cases leads to the loss of the connection between the controlled and the controlling parameters, which eliminates the possibility of optimization using the given model. In this connection the development of approximate methods of optimization for a system of equations with variable coefficients is an urgent task.

It is known that the drying process in the general case is described by a system of differential equations of conjugate heat and mass transfer proposed by A.V. Lykov [2] in which the coefficients to the partial derivatives are combinations of the thermophysical and thermodynamic characteristics of the material being dried. For each concrete process they are different functions of the principal drying parameters: the temperature and moisture content.

By replacing these functions by constants we obtain a new simplified system of equations with constant coefficients, the solution of which is considerably simplified.

We will distinguish a class of these simplified systems of equations, each of which has a solution, and for one of the systems we will find an approximate method of solution of the problem of the optimum control of drying processes.

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Let us examine the main idea of the method for a one-dimensional system. We will consider a nonsteady process of convective drying with transverse supply of the drying agent relative to the motion of the material. The parameters of the material and the drying agent are distributed along the x coordinates along the motion of the material and depend on the time τ.

As the initial system we take the system of differential equations of heat and mass transfer in the absence of an overall pressure gradient [1]:

$$\left.\begin{aligned}\frac{\partial u}{\partial \tau} &= k_{11}\nabla^2 u + k_{12}\nabla^2 T\\ \frac{\partial u}{\partial \tau} &= k_{21}\nabla^2 u + k_{22} T\end{aligned}\right\}. \tag{1}$$

The coefficients to the partial derivatives in the boundary conditions are also replaced by constants in the construction of the simplified system [2]. As the controlled parameters we take the moisture content u of the material and its temperature T, and as the controlling parameter we take the temperature α of the drying agent, which we will seek as a function of the time τ and the coordinate x, which varies in the region $0 \le x \le l$. The coefficients of the system (1) and of the boundary conditions are functions of the coordinates and of the parameters entering into the system:

$$k = k(\tau,\ x,\ u,\ \alpha). \tag{2}$$

Certain limitations are imposed on the parameters of the system in accordance with technological requirements:

$$u \in X, \quad T \in Y, \quad \alpha \in U. \tag{3}$$

The criterion of optimality is written in the form of an integral functional:

$$I = \int_0^l \int_0^{\tau^*} F\left(\tau,\ x,\ u,\ T,\ \frac{\partial u}{\partial x},\ \frac{\partial u}{\partial \tau},\ \frac{\partial T}{\partial x},\ \frac{\partial T}{\partial \tau},\ \alpha,\ d\tau,\ dx\right), \tag{4}$$

where τ^* is the drying time.

We divide the segment $[0, l]$ into n equal parts $0 < h < 2h < \ldots < nh = l$ and the segment $[0, \tau^*]$ into m parts $0 < h_1 < 2h_1 < \ldots < mh_1 = \tau^*$. In the process the space–time region $G = [0 \le x \le l,\ 0 \le \tau \le \tau^*]$ is divided into subregions ΔG_s $(s = 1, \ldots, nm)$, which we number as follows: Of two regions the one with the later time segment will be subsequent, while for equal time intervals the region whose spatial interval is farther from the origin of coordinates is subsequent.

We formulate the problem of optimization as follows: Among all possible controls $\alpha(\tau, x)$ to find that for which the corresponding solution of the system of equations (1) of the mathematical description provides for the minimum of the functional I and for which the limitations (3) are observed. In this case the control α is assumed to be constant in each region ΔG, which provides for the zone-wise supply of drying agent, the amount of which is changed either in the transition from one spatial region to another or within the limits of one region but differ some time determined by the partition made above. Since the system (1) belongs to the selected class, the system with constant coefficients has a solution. Let

$$\left.\begin{aligned}u &= \varphi_1(\tau,\ x,\ \bar{k})\\ T &= \varphi_2(\tau,\ x,\ \bar{k})\end{aligned}\right\} \tag{5}$$

be a solution of the simplified system, where its entire set of coefficients is designated through $\bar{k}$.

Starting from the formal solution (5) and Eqs. (2), we find an approximate solution of the system (1) relative to u and T over the entire space–time region, and we also derive the recurrent equations needed later. Let us consider the rectangle ΔG_1, to the sides of which the segments $[0, h]$ of the straight line $\tau = 0$ and $[0, h_1]$ of the straight line $x = 0$ belong and in which the values of u and T are known from the boundary conditions. By integrating we find $u^{(0)}$ and $T^{(0)}$, the mean values of u and T over the set of indicated segments, and substitute them into Eqs. (2). The approximate values of the coefficients of the system (1) which are obtained take the form

$$k^{(1)} = k^{(1)}(\tau,\ x,\ u^{(0)},\ T^{(0)},\ \alpha^{(1)});\ (\tau,\ x) \in \Delta G_1, \tag{6}$$

where $\alpha^{(1)}$ denotes the as yet unknown control α in the region ΔG_1. From the construction of the simplified system it is seen that some function from the set (2), and consequently from (6), corresponds uniquely to each of its coefficients. Replacing each coefficient of k in Eqs. (5) by the corresponding function from the set (6), we obtain the approximate equations for u and T in the region ΔG_1:

$$\left.\begin{aligned} u &= \Psi_1^{(1)}(\tau,\ x,\ u^{(0)},\ T^{(0)},\ \alpha^{(1)}) \\ T &= \Psi_2^{(1)}(\tau,\ x,\ u^{(0)},\ T^{(0)},\ \alpha^{(1)}) \end{aligned}\right\}. \tag{7}$$

Since $\alpha^{(1)}$ is constant in ΔG_1, from (7) one can determine $u^{(1)}$ and $T^{(1)}$, the mean values of u and T in the region ΔG_1, from the equations

$$u^{(1)} \frac{1}{hh_1}\int_0^{h_1}\int_0^{h}\Psi_1^{(1)}(\tau,\ x,\ u^{(0)},\ T^{(0)},\ \alpha^{(1)})\,d\tau\,dx,$$

$$T^{(1)} = \frac{1}{hh_1}\int_0^{h_1}\int_0^{h}\Psi_2^{(1)}(\tau,\ x,\ u^{(0)},\ T^{(0)},\ \alpha^{(1)})\,d\tau\,dx.$$

Thus

$$\left.\begin{aligned} u^{(1)} &= f_1^{(1)}(u^{(0)},\ T^{(0)},\ \alpha^{(1)}) \\ T^{(1)} &= f_2^{(1)}(u^{(0)},\ T^{(0)},\ \alpha^{(1)}) \end{aligned}\right\}. \tag{8}$$

By analogy with Eqs. (6), using the boundary conditions for the region ΔG_1, we obtain the equations

$$k^{(2)} = k^{(2)}(\tau,\ x,\ u^{(1)},\ T^{(1)},\ \alpha^{(2)});\ (\tau,\ x)\in\Delta G_2,$$

where $\alpha^{(2)}$ is the control in the region ΔG_2.

Following the same reasoning as for the region ΔG_1, we arrive at equations analogous to (7) and (8). By extending this process further, we obviously arrive at recurrent equations valid for each of the regions:

$$\left.\begin{aligned} &u = \Psi_1^{(s)}(\tau,\ x,\ u^{(s-1)},\ T^{(s-1)},\ \alpha^{(s)}) \\ &T = \Psi_2^{(s)}(\tau,\ x,\ u^{(s-1)},\ T^{(s-1)},\ \alpha^{(s)}) \\ &s = 1,\ \ldots,\ nh;\quad (x,\ \tau)\in\Delta G_s \end{aligned}\right\}, \tag{9}$$

$$\left.\begin{aligned} u^{(s)} &= f_1^{(s)}(u^{(s-1)},\ T^{(s-1)},\ \alpha^{(s)}) \\ T^{(s)} &= f_2^{(s)}(u^{(s-1)},\ T^{(s-1)},\ \alpha^{(s)}) \end{aligned}\right\}. \tag{10}$$

It should be noted that while $u^{(s)}$ and $T^{(s)}$ are constants, their values essentially depend on $\alpha^{(s)}$ and can take on one or another value depending on the value of the latter.

Let us consider the functional (4). Obviously,

$$I = \sum_{s=1}^{N}\int_{\Delta G_s} F\left(\tau,\ x,\ u,\ T,\ \frac{\partial u}{\partial\tau},\ \frac{\partial u}{\partial x},\ \frac{\partial T}{\partial\tau},\ \frac{\partial T}{\partial x},\ \alpha\right)d\tau\,dx,$$

where N = nh.

Substituting the values of u and T from Eqs. (9) into the integrand, we obtain

$$I = \sum_{s=1}^{N}\int_{\Delta G_s} F\left(\tau,\ x,\ \Psi_1^{(s)},\ \Psi_2^{(s)}\ \frac{\partial u}{\partial x},\ \frac{\partial u}{\partial\tau},\ \frac{\partial T}{\partial x},\ \frac{\partial T}{\partial\tau},\ \alpha\right)d\tau\,dx.$$

Calculating the integral, we arrive at the following sum:

$$I = R_N\sum_{s=1}^{N} r_s(u^{(s-1)},\ T^{(s-1)},\ \alpha^{(s)}). \tag{11}$$

Thus, the problem of the optimum spatial distribution of the temperature of the drying agent over the drying chamber has been reduced to the problem of the search for the optimum control of a multistage process for which the system (10) serves as the mathematical description and the sum (11) is the optimality criterion, with the limitations (3) on the parameters. For the solution of problems of the optimization of multistage processes the most effective is the method of dynamic programming, using which the given problem is solved completely [3], a result of which is the obtainment of the optimum zone-wise distribution $\alpha^{(s)}_{opt}$ (s = 1, . . . , N) of the temperature of the drying agent. The accuracy of the cited method of solution can be considered as sufficient when the quantization intervals are small [4].

In conclusion, we note that with a substantial N the given method requires great computational work in which uniform calculations predominate, which is convenient for the use of computers. A block diagram of an algorithm for the optimization of multistage processes by the method of dynamic programming, which is realizable on a computer, is presented in [3].

Let us illustrate what has been presented on a concrete example. With some simplifications the process of drying of a moving layer of disperse material in the form of infinite cylinders of radius R moving in a continuous stream is described by the following system of differential equations [5]:

$$\left.\begin{aligned} &\frac{\partial u}{\partial \tau}+\omega\frac{\partial u}{\partial x}-a_m\left(\frac{\partial^2 u}{\partial^2 r}+\frac{1}{r}\frac{\partial u}{\partial r}\right)=0\\ &\frac{\partial T}{\partial \tau}+\omega\frac{\partial u}{\partial x}+\frac{A}{c\gamma}(T-\alpha)-f_d\left(\frac{\partial \bar{u}}{\partial \tau}+\omega\frac{\partial \bar{u}}{\partial x}\right)=0\end{aligned}\right\}, \tag{12}$$

where

$$\bar{u}(\tau, x)=\frac{2}{R^2}\int_0^R ru(\tau, x, r)\,dr,$$

with the boundary conditions

$$\left.\begin{aligned} &u(\tau, 0, r)=u^{(0)}(\tau, r),\\ &\frac{\partial u}{\partial \tau}(\tau, x, R)=-\frac{B}{a_m}(u_R-u_e)\\ &\frac{\partial u}{\partial r}(\tau, x, 0)=0\ \text{(symmetry condition)}\\ &T(\tau, 0)=f_1(\tau)\end{aligned}\right| \tag{13}$$

and the initial conditions

$$\begin{aligned} &u=(0, x, r)=u_0(x, r)\\ &T=(0, x)=f_2(x).\end{aligned}$$

Here r is the coordinate from the axis of symmetry of a cylinder to its surface ($0 \le r \le R$), x is the coordinate along the drying chamber ($0 \le x \le l$), and $u_R = u(\tau, x, R)$. All the coefficients in (12) and (13) are functions of the type of (2). The following limitations are imposed on the parameters: $T(\tau, x) \le T_{max}$ = const,

$$\left.\begin{aligned} &u(\tau, l)=u_{out}(\tau)\ \text{is the output moisture content,}\\ &\alpha_{min}\leqslant\alpha(\tau, x)\leqslant\alpha_{max}\ (\alpha_{min}, \alpha_{max}\ \text{are constants})\end{aligned}\right\}. \tag{14}$$

We formulate the optimization problem as follows: Among all possible distributions of the temperature of the drying agent $\alpha(\tau, x)$ to find that for which the amount of heat supplied to the drying chamber is minimal for a given regime of output moisture content. With a constant flow rate of drying agent the amount of heat is proportional to the temperature of the heat-transfer agent coming from the air heater. Consequently, the functional which must be minimized has the form

$$I=\int_0^\tau\int_0^l \alpha(\tau, x)\,d\tau\,dx. \tag{15}$$

Since letter designations are presented in place of functional expressions for the coefficients of the system (12), the system itself can serve as its simplified system with constant coefficients if the coefficients in it and in the boundary conditions are taken as constants. Let us find the solution of the simplified system.

Let us consider the first equation of the system (12). We set $u(\tau, x, r) = u_e + v(\tau, x, r)$ and substitute it into the equation and the boundary conditions. As a result of the change we obtain boundary conditions allowing us to use the Fourier method [7] for the solution of this equation. Solving by this method, we obtain

$$u(\tau, x, r)=u_e+\sum_{n=1}^{\infty}\exp\left\{-\frac{\lambda_n^2 a_m}{\omega}x\right\}\theta_n\left(\tau-\frac{x}{\omega}\right)\Phi_n(r), \tag{16}$$

where

$$\theta_n(\tau) = \int_0^k [u^0(\tau, r) - u_e] r J_0^2(\lambda, r)\, dr,$$

$$\Phi_n(r) = \frac{J_0(\lambda, r)}{N_n^2}, \text{ where } N_n^2 = \int_0^R r J_0^2(\lambda_n r)\, dr,$$

while $J_0(x)$ is a zeroth-order Bessel function of the first kind; $\lambda_n = \mu_n/R$, where μ_n are the roots of the equation

$$BRJ_0(\mu) + a_m \mu J_0^1(\mu) = 0.$$

From (16) we find

$$u(\tau, x) = u_e + \frac{2}{R} \sum_{n=1}^{\infty} \frac{J_2(\lambda_n R)}{\lambda_n N_n^2} \theta_n\left(\tau - \frac{x}{\omega}\right) \exp\left\{-\frac{\lambda_n^2 a_m}{\omega} x\right\}. \tag{17}$$

The second equation of the system (12) represents the Cauchy problem, the solution of which is the function

$$\begin{aligned} T(\tau, x) = &\left(1 - \exp\left\{\frac{A}{c\gamma\omega} x\right\}\alpha\right. \\ &+ \frac{2Ba_m}{R} \sum_{n=1}^{\infty} \frac{\lambda_n J_1(\lambda_n R)}{\left(-\frac{A}{\gamma} + \lambda_n^2 a_m c\right) N^2} \\ &\times \theta_n\left(r - \frac{x}{\omega}\right)\left(\exp\left\{-\frac{\lambda_n^2 a_m}{\omega} x\right\} - \exp\left\{\frac{A}{c\gamma\omega} x\right\}\right) \\ &+ \exp\left\{-\frac{A}{c\gamma\omega} x\right\} f_1\left(\tau - \frac{x}{\omega}\right). \end{aligned} \tag{18}$$

The solutions (17) and (18) will later play the role of Eqs. (5).

We reduce the problem to a form allowing us to apply the method of dynamic programming. We divide the region $G = [0 \le \tau \le \tau^*,\ 0 \le x \le l]$ into subregions ΔG_s ($s = 1, \dots, nm$), as was done above. The values of the coefficients in the region ΔG_1 are found on the basis of the values of $u^{(0)}$ and $T^{(0)}$, determined from the equations

$$u^{(0)} = \frac{2}{R^2(h - h_1)} \int_0^h \int_0^R u_0(\tau, x)\, r dr dx + \int_0^h \int_0^R u^0(\tau, r)\, r dr d\tau,$$

$$T^{(0)} = \frac{1}{h + h_1}\left(\int_0^h f_2(x)\, dx + \int_0^{h_1} f_1(\tau)\, d\tau\right).$$

By substituting the values of the coefficients into Eqs. (17) and (18) we find the solutions $u(\tau, x)$ and $T(\tau, x)$ in the region ΔG_1, by integrating which we obtain $u^{(1)}$ and $T^{(1)}$, and so forth, until equations of the type of (9) and (10) are obtained in the entire region G. Since the $\alpha(\tau, x)$ in each of the ΔG_s do not depend on τ and k, we have

$$I = \int_0^{\tau^*} \int_0^l \alpha(\tau, x)\, d\tau dx = hh_1 \sum_{s=1}^{N} \alpha^{(s)}.$$

The equations obtained fully describe the multistage process, and by applying the method of dynamic programming to it we obtain the optimum zone-wise distribution of the temperature $\alpha(\tau, x)$ of the drying agent.

NOTATION

u, moisture content of moist solid, kg/kg; T, temperature of material being dried, °C; τ, time, sec; a_m, coefficient of moisture diffusion, m^2/sec; A, B, empirical coefficients for moving layer; c, specific heat capacity of drying agent, J/(kg · °K); ω, average velocity of center of mass of disperse medium, m/sec;

u_e, equilibrium moisture content of material, kg/kg; l, length of drying chamber, m; γ, concentration of dry substance in moist disperse material, kg/kg.

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EFFICIENCY OF COOLING THERMOELECTRIC ELEMENTS OF ARBITRARY SHAPE

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The problem of the limiting efficiency of thermoelectric cooling is considered in the general case when no limitations are imposed on the shape of the thermoelectric elements and their contact surfaces.

It is well known that the permissible temperature drop and the limiting power efficiency of thermoelectric elements of prismatic shape are uniquely determined by the figure of merit of the thermoelectric materials and the temperature level at which the elements operate and are independent of their geometrical dimensions [1]. It is of considerable interest to clarify what this behavior is in the general case when no limitations are imposed on the shape of the thermoelectric element and on its contact surfaces.

Consider a thermoelectric element (see Fig. 1) having two contact surfaces s_0 and s_1. We will assume that the heat exchange between the thermoelectric element and the external sources only occurs over the surfaces of the contacts, which are simultaneously isothermal and equipotential, while the remaining surface of the thermoelectric element is adiabatically and electrically insulated. We will consider the properties of the temperature field which is established when a potential difference u_1-u_0 is applied, and we will determine the heat flow entering the contact surfaces along the body of the thermoelectric element.

If we ignore the temperature dependence of the physical parameters of the thermoelectric material, the temperature field inside the region v bounded by the surface s of the thermoelectric element corresponds to the Poisson equation

$$\nabla^2\vartheta = -\frac{i^2}{\lambda\sigma}. \tag{1}$$

Equation (1) is uniform and there are also the nonuniform boundary conditions:

$$\vartheta|_{s_0} = 0; \quad \vartheta|_{s_1} = T_1 - T_0; \quad \left.\frac{\partial\vartheta}{\partial n}\right|_{\Sigma} = 0. \tag{2}$$

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